Math 347 Worksheet

Worksheet 13: divisibility properties

November 28, 2018

1) Consider the set $\mathbb{Z}$. For each of the following relations determine if it defines an equivalence relation[1]:

   (i) $P = \mathbb{Z} \times \mathbb{Z}$;

   (ii) $P = \mathbb{Z} \times \{0\}$;

   (iii) $P = \{(a, a) \mid a \in \mathbb{Z}\}$;

   (iv) $P = \{(a, b) \in \mathbb{Z}^2 \mid a \geq b\}$;

   (v) $P = \{(a, b) \in \mathbb{Z}^2 \mid a - b = 3 \cdot k, \text{ for some } k\}$.

2) Determine if the following functions are injective or surjective.

   (i) $3\cdot : \mathbb{Z}/5\mathbb{Z} \to \mathbb{Z}/5\mathbb{Z}$;

   (ii) $2\cdot : \mathbb{Z}/4\mathbb{Z} \to \mathbb{Z}/4\mathbb{Z}$;

   (iii) $5\cdot : \mathbb{Z}/6\mathbb{Z} \to \mathbb{Z}/6\mathbb{Z}$.

3) Consider the function $f : \mathbb{Z}/n\mathbb{Z} \to \mathbb{Z}/n\mathbb{Z}$ given by $f(x) = x^2$. For which n is f injective?

4) Prove that the first six powers of 10 belong to different congruence classes modulo 7.

[1] Recall that one needs to check that R is reflexive, symmetric and transitive.